To Be More Concrete About Abstracta and Concreta

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Abstract. The distinction between abstract and concrete entities, especially its precise characterization, remains underexplored in foundational ontology research. This paper aims to constitute the initial steps towards a formal ontology of abstracta and concreta. We begin by presenting three existing criteria (epistemic, spatiotemporal and causal) for the abstract/concrete distinction. We illustrate them with some well-known upper ontologies. After examining the shared assumption by the three criteria that any entity is either abstract or concrete but not both, we develop an alternative and more general framework for formally representing abstract and concrete entities. In particular, we propose a relational account of them by introducing the relation of "concretization". The pivotal idea is that being abstract (or concrete) amounts to being concretized by (or concretizing) some other entity. We also briefly discuss a concretization-based reinterpretation of the spatiotemporal criterion.

Keywords. abstract, concrete, concretization, spacetime, causality, upper ontology

1. Introduction

1.1. Background

Abstract and concrete entities constitute an important categorical distinction in the field of formal ontology. Borgo et al. [1] list this distinction ("Concrete vs Abstract") as one of the philosophical issues that are relevant to modern work in foundational ontology, as well as "Continuants vs Occurrents", "Universals vs Particulars", and "Substance vs Accident" (which is synonymous with the distinction between independent and dependent entities). To borrow some paradigmatic examples, numbers, sets and propositions are abstract; whereas molecules, people and planets are concrete.

The distinction between abstracta and concreta nonetheless remains largely unexplored in foundational ontology research. For instance, Borgo et al. [1] state:

It is common to distinguish between concrete entities such as continuants and occurrents, which exist in space and/or time, and abstract entities which, if they can

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be said to exist at all, do so outside space and time, prototypical examples being mathematical entities such as numbers, functions, and sets. [1, p. 6]

According to this spatiotemporal criterion for the abstract/concrete distinction, an entity is abstract just in case it exists outside space and time (or spacetime) and it is otherwise concrete. Consider now so-called "impure sets" such as $\{Earth\}$, i.e. the set whose urelement is the planet Earth. The impure set $\{Earth\}$ can be classified as abstract because sets are "prototypical examples" of abstract entities by the spatiotemporal criterion, as they exist outside space and time. But it may also be considered as concrete because, at any time at which it exists (whether eternally or when Earth exists), $\{Earth\}$ seems to exist only in space where Earth exists. Does the impure set $\{Earth\}$ clearly belong to either category, according to the spatiotemporal criterion? More generally, is it reasonable to assume that any entity is either abstract or concrete but not both?

This paper aims to address such fundamental questions about abstract and concrete entities in order to lay the groundwork for a full-fledged ontology of them. The paper is organized as follows. The remainder of Section 1 specifies the scope of our investigation. Section 2 presents three existing criteria for the abstract/concrete distinction: epistemic, spatiotemporal and causal. We also illustrate them with three major upper ontologies: Basic Formal Ontology (BFO), a Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE) and the General Formal Ontology (GFO). Section 3 scrutinizes the shared assumption by the three criteria that any entity is either abstract or concrete but not both. Then we develop an alternative and more general framework for formally representing abstracta and concreta. More specifically, we propose a relational account of them by introducing the relation of "concretization". The central idea is that being abstract (respectively: concrete) amounts to being concretized by (respectively: concretizing) some other entity. Section 4 offers the discussion. Section 5 concludes the paper.

1.2. Scope

We will articulate the scope of our study of abstract and concrete entities. Firstly, discussion on the abstract/concrete distinction has a long history in philosophy (see e.g. [2]). In particular, there is a traditional debate over the existence of abstract entities between platonism (which argues for it) and nominalism (which denies it). There are also many forms of platonism and nominalism: e.g. nominalism about mathematical entities such as numbers and sets. Since the purpose of this paper is to investigate the abstract/concrete distinction in the context of formal ontology, we will postulate that this distinction is ontologically substantive and leave aside eliminativism about it (as endorsed by some nominalists). Effectively, as we will see in Section 2.2, major upper ontologies generally recognize (explicitly or implicitly) the importance of the abstract/concrete distinction.

Secondly and relatedly, we will remain neutral as to the role of abstract entities in ontology of natural language. Although being assumed in this paper, the ontological substantivity of the abstract/concrete distinction may not necessarily be supported by apparent reference to abstract objects in ordinary discourse. For instance, Moltmann [3, 4] hypothesizes that natural language does not involve reference to abstract objects in its "ontological core", but only its "ontological periphery" — where, intuitively: "the ontological core consists in expressions or uses of expressions not involving ontological reflection and the ontological periphery consist of expressions or uses of them involving ontological reflection" [4, p. 256].

Thirdly and finally, there are various ways of drawing the distinction between abstract and concrete entities, as they are categorized and examined by Lewis [5, pp. 81-86]. Among these diverse approaches, we will focus on what Lewis calls "the way of negation": a way of specifying the abstract/concrete distinction by defining abstracta as entities that *lack* some salient features (e.g. spatiotemporal location) of concreta. For one thing, as Falguera et al. [2] point out, this negation-based approach to abstract and concrete entities is adopted by many explicit characterizations of them in the literature. For another, it will help to understand different views of the abstract/concrete distinction which are embraced by different upper ontologies (see Section 2.2).

2. Abstracta and Concreta: A State-of-the-Art Overview

2.1. *Three Existing Criteria: Epistemic, Spatiotemporal and Causal*

We will present three existing criteria for characterizing abstract and concrete entities — among several existing ones based on "the way of negation" (see e.g. [2]). As will be illustrated in Section 2.2, these selected criteria will be particularly useful in comparing some prominent upper ontologies *vis-à-vis* the abstract/concrete distinction.

Firstly, the *epistemic criterion* identifies being abstract with being *non*-physical, or more precisely being mental, and being concrete with being physical:

The epistemic criterion for abstracta and concreta
An entity is *epistemically abstract* if and only if it is mind-dependent (synonym: "mental"): that is, it depends (existentially) on some mental process.[2]
An entity is *epistemically concrete* if and only if it is mind-independent (synonym: "physical"): that is, it does not depend on any mental processes.

To take some canonical examples, Earth is epistemically concrete because it would continue to exist even if there is no mental process in the world. The empty set $\emptyset$ is epistemically abstract, by contrast, because it can be considered as the construction of some mental process, as mathematical intuitionists/constructivists would argue (see e.g. [6]). It is interesting to note that this criterion is closely related to "the way of abstraction" in Lewis's [5] terms — according to which an entity is abstract if (and only if) it is the referent of *abstraction*: a mental process of conceptualizing some common features of certain entities while omitting other irrelevant features thereof (cf. [7]).

Secondly, the *spatiotemporal criterion* — which says that abstracta are *non*-spatiotemporal — is one of the standard ways of understanding the abstract/concrete distinction, as it is mentioned by Borgo et al. [1] (see Section 1.1):

The spatiotemporal criterion for abstracta and concreta
An entity is *spatiotemporally abstract* if and only if it exists outside spacetime (or alternatively: it exists outside space and time).
An entity is *spatiotemporally concrete* if and only if it exists in spacetime (or alternatively: it exists in space and/or time).

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[2]In using the term "process" in this paper, we remain uncommitted to any specific ontologies of occurrents/perdurants, especially the ones which draw a sharp distinction between processes and events.

For instance, Earth is spatiotemporally concrete and the empty set $\emptyset$ is spatiotemporally abstract, as the former exists in spacetime but the latter exists nowhere and "no when" in the sense of "outside time". It is important to remark on the terms "spacetime", "space" and "time" therein. We formulated the spatiotemporal criterion primarily with the term "spacetime" because there is no unique separation of spacetime into spatial and temporal components, according to modern physics — or at least when it comes to the non-Newtonian spacetime structure (cf. [8]). But we also indicated an alternative formulation of this criterion based on the terms "space" and "time" (see e.g. [1] and also Section 1.1), especially because not all upper ontologies intend to capture the notions of space and time as conceived in modern physics (see e.g. [9] and also Section 2.2).

Thirdly, the *causal criterion* — which says that abstracta are *non*-causal — is an equally popular approach to abstract and concrete entities:

> **The causal criterion for abstracta and concreta**
> An entity is *causally abstract* if and only if it is causally inefficacious.
> An entity is *causally concrete* if and only if it is causally efficacious.

By way of illustration, Earth is causally concrete, as it comes into being *as a result* of a mixture of dust and gas around the sun; while the empty set $\emptyset$ is causally abstract, as it can make no causal contribution to the world. Note that a more rigorous formulation of this criterion will require a well-developed account of causality which can clarify the term "causally (in)efficacious" therein, although the notion of causality *per se* is notoriously difficult to analyze from an ontological point of view (see e.g. [10]).

2.2. Abstracta and Concreta in Upper Ontologies: A Case-Study

We will survey the treatment of abstract and concrete entities in three major upper ontologies — BFO, DOLCE and GFO — according to the three criteria (epistemic, spatiotemporal and causal) for their distinction. During the course of the survey, we will also consider the ontological status of space and time (or spacetime), in particular the issue of whether and by which criterion they are abstract or concrete, because it is inextricably connected with the spatiotemporal criterion.

2.2.1. Basic Formal Ontology (BFO)

Basic Formal Ontology (BFO) [11,12,13] does not have a category of abstracts in its taxonomy, as Borgo et al. [1] point it out. Indeed, regarding BFO 2.0 (as of 2015), Smith et al. [11, p. 6] state: "We leave open the question of how, if at all, BFO would deal with numbers, sets, and other mathematical entities, and with propositions (conceived in the sense of ideal meanings)." A comprehensive treatment of such typical abstract entities as numbers, sets and propositions seems to remain a future task for BFO up to date.

Arguably, one reason why BFO should remain neutral as to abstract entities is that BFO may subscribe to the spatiotemporal criterion.[3] In BFO, particulars are classified into either continuants or occurrents and they reflect, respectively, a purely spatial

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[3]An alternative interpretation would be that BFO embraces the epistemic criterion and abstract entities are out of the scope of BFO because BFO is theoretically underpinned by the methodology of "ontological realism" [14] according to which ontologies should represent actual entities, as described by science. We set aside this interpretation partly because it will require justification for the vital explanatory role of such paradigmatic abstracta as numbers and sets in modern science.

("SNAP") view of the world — which represents a series of instantaneous *snap*shots of reality — and a purely spatiotemporal ("SPAN") view of the world — which represents changes within time *span*s in reality [15]. By the spatiotemporal criterion, there is no abstract entity at the level of particulars in BFO because continuants and occurrents are spatial and spatiotemporal entities, respectively.

Moreover, BFO may also be committed to the causal criterion. To see this, let us focus on the usage of the term "abstract" for explaining *generically dependent continuants* in BFO.[4] A generically dependent continuant can be considered as a "complex continuant pattern[s] either of the sort created by authors or designers or [...] brought into being through the processes of evolution" [12, p. 105]. Examples include information artifacts (e.g. the novel *Robinson Crusoe*) and biological sequences (e.g. DNA sequences).

A generically dependent continuant is described as an "abstract pattern" and exists only if it is *concretized* in some other kind of entity: for instance, "The novel *Robinson Crusoe* is a generically dependent continuant instance, an *abstract pattern*" [12, p. 106] and it is concretized in "a certain complex quality (of a certain quantity of bound paper and associated small piles of printer's ink)" [12, p. 105] of a book of the novel.

We can interpret the abstract-ness of generically dependent continuants according to the causal criterion. For example, the novel *Robinson Crusoe* might seem to have some causal efficacy (e.g. to affect the reader). But it is rather its concretizations (e.g. ink pattern) that have such causal efficacy and the novel *Robinson Crusoe* itself is causally inefficacious. (It must be emphasized that so-called "abstract artifacts" such as the novel *Robinson Crusoe* may be difficult to classify clearly as either abstract or concrete by the causal criterion. We will address this issue in Section 3.1.)

As for space and time in BFO, spatial regions are continuants; while *temporal regions* and *spatiotemporal regions* are occurrents (see e.g. [8,9] for detailed discussion). As Borgo et al. [1, p. 6] say, in BFO: "times and places [...] are regarded as concrete, times being occurrents and places continuants." More accurately, space and time in BFO would be spatiotemporally concrete, as they should be physically taken because of BFO's realist methodology [14] (see Footnote 3 for details on this methodology).

2.2.2. A Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE)

A Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE) [16,17,18] has a category of abstracts in its taxonomy, as Borgo et al. [1] say. In DOLCE, *abstracts* constitute one of the four top-level categories as well as *endurants*, *perdurants*, and *(individual) qualities*. DOLCE explicitly adopts the spatiotemproal criterion: "*abstract[s] entities* exist neither in space nor in time, i.e. they are not localized. On the other hand, *concrete entities* (or *concretes*) are defined as entities that exist at least in time" [16, p. 10]. For that matter, DOLCE characterizes spatial and temporal locations as qualities such as colors and weights (see e.g. [9,19] for detailed discussion) and Masolo et al. [16, p. 19] say: "The main characteristic of abstract entities [in DOLCE] is that they do not have spatial nor temporal qualities, and they are not qualities themselves."

At the same time, DOLCE may also embrace some kind of epistemic criterion.[5] In DOLCE, one subtype of qualities are *abstract qualities*. An abstract quality is a quality

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[4]We will occasionally italicize names for the entities in a given ontology (as in BFO) for the sake of clarity.

[5]Note that Masolo et al. [16, p. 10] touch on the causal criterion ("abstracts possess no causal power while concretes do") but they say: "In what follows, we focus on the [spatiotemporal] characterization of abstracts." We take this text to mean that DOLCE does not endorse the causal criterion.

that directly inheres in a *non-physical endurant*: an endurant that does not have a direct spatial quality. To borrow Masolo et al.'s [16, p. 15] example, "the value of an asset" is an abstract quality. They also state: "this terminology [about "abstract quality"] is very problematic: for instance, it should be clear that abstract qualities are *not* abstracts, since they have a temporal location. Better suggestions are welcome." [16, p. 25]

One possible interpretation for the term "abstract quality" would be that abstract qualities are not spatiotemporally abstract but their abstract-ness may be (at least partially) understood according to the epistemic criterion. In DOLCE, there are at least two subtypes of non-physical endurants: *mental objects* and *social objects*. A mental object (e.g. a private experience) would dependent on an individual agent and a social object (e.g. an asset) is "generically dependent [on] a community of agents" [16, p. 24]. Given that agents have an intentional dimension, abstract qualities of mental or social objects (or perhaps of a wider range of non-physical endurants) would be epistemically abstract.

As for space and time in DOLCE, *temporal regions* and *space regions* are *(quality) regions*, which are in turn abstracts. In this respect, DOLCE "treat[s] times and places (...) as abstract entities" [1, p. 6]. To put it more precisely, space and time in DOLCE can be spatiotemporally or epistemically abstract because DOLCE aims "to capture the intuitive and cognitive bias underlying commonsense" [17, p. 279] and they can understood cognitively as well as physically.

2.2.3. The General Formal Ontology (GFO)

The General Formal Ontology (GFO) [20,21,22] employs the term "abstract" in several ways, some of which will be presented for the purpose of this paper. In the first place, GFO exhibits a three-layered meta-ontological structure and consists of three corresponding ontologies: the abstract top ontology, the abstract core ontology and the (foundational) basic ontology. The abstract top ontology consists of only two "meta-meta-categories", *sets* and *items*, and it can be set-theoretically represented.

The abstract core ontology, by contrast, consists of categories and relations that can serve as meta-level entities for the categories associated with the basic ontology. Examples include *categories* and *individuals*, the former being instantiated by the latter. For our current goal, it is useful to see Herre et al.'s [20] discussion on Sowa's [23] distinction between *physical entities* and *abstract entities* — which is based on the spatiotemporal criterion — in his top-level ontology. For Sowa (as explained by Herre et al.), a physical entity bears the *representation* or *characterization* relation towards an abstract entity. To take his example, this particular physical encoding in a physical medium characterizes a piece of information (which is an abstract entity). Herre et al. [20, p. 74] remark: "Intuitively, the notions of physical and abstract entities [in Sowa's ontology] correspond to the GFO notions of individual and category, respectively."

Concerning the basic ontology, GFO utilizes the abstract/concrete distinction within its tripartite classification of individuals: *space-time entities*, *concrete individuals*, *abstract individuals*. Examples of space-time entities include *space regions* and *choronids*: roughly, time intervals with different end points. As regards concrete and abstract individuals, Loebe et al. [22, p. 75] explain: "*Abstract individuals* are independent of space and time, for example the number π, whereas *concrete individuals* have an immediate relation to time or to time and space."

GFO provides multiple different perspectives on abstract entities. First, GFO is explicitly committed to the spatiotemporal criterion, as is shown by its explanation of ab-

stract and concrete individuals. Second, categories and individuals in GFO can be considered, respectively, as abstract and physical entities in Sowa's [23] sense: in particular, individuals can be said to represent or characterize (in Sowa's terms) categories. Third, sets have a special status in GFO. Numbers are abstract individuals but sets are not (directly, at least) classified as abstract or concrete in GFO, although both numbers and sets are generally reckoned to be abstract in the literature.

As for space and time in GFO, there can be at least two interpretations for their ontological nature. One interpretation is that they are spatiotemporally neither abstract nor concrete because space-time entities (in GFO's sense) are distinct from both abstract and concrete individuals. The idea may be that space and time constitute a criterion for drawing the abstract/concrete distinction, but they *per se* cannot be classified as abstract or concrete. The other interpretation is that space and time in GFO are epistemically abstract because they are abstractions of the continuum that can be accessed through introspection and they are thus "phenomenal space" and "phenomenal time", respectively.

Table 1 summarizes our survey of the treatment of abstract and concrete entities in BFO, DOLCE and GFO, based on the three existing criteria for distinguishing them:

Table 1. Summary of the upper ontologies BFO, DOLCE and GFO with respect to abstracta and concreta

commitment	BFO	DOLCE	GFO
epistemic criterion		X	
spatiotemporal criterion	X	X	X
causal criterion	X		
space and time: abstract or concrete?	spatiotemporally concrete	spatiotemporally or epistemically abstract	1) spatiotemporally neither 2) epistemically abstract

3. A Relational Account of Abstracta and Concreta

3.1. Exhaustivity and Exclusivity: Underlying Assumptions Behind the Three Criteria

Let us now examine the three existing criteria — epistemic, spatiotemporal and causal — for distinguishing between abstract and concrete entities. As we saw in Section 2.1, the planet Earth and the empty set $\emptyset$ are categorized, respectively, as concrete and abstract according to any of the three criteria. It may be nonetheless controversial whether all entities can be classified so clearly as either abstract or concrete by these criteria.

To illustrate this point, consider how (1) impure sets (e.g. $\{Earth\}$) and (2) abstract artifacts (e.g. the novel *Robinson Crusoe*) can be interpreted as (A) abstract or (C) concrete according to each of the three criteria. Note that we owe parts of the following analysis to Falguera et al.'s [2] discussion:

- *The epistemic criterion*
  * (1-A) Impure sets are abstract because all sets (whether pure or impure) are constructions of some mental process. (1-C) Some impure sets are abstract and others are concrete: e.g. $\{An\_afterimage\_of\_Earth\}$ is abstract because an afterimage of Earth depends on some mental process but $\{Earth\}$ is concrete because Earth does not depend on any mental processes.

    ∗ (2-A) Abstract artifacts are abstract, as the novel *Robinson Crusoe* depends on Daniel Defoe's mental process or on the subsequent readers'. (2-C) Abstract artifacts are concrete, as the novel *Robinson Crusoe* can exist even if nobody is thinking about it at any time (i.e. it does not depend on any mental processes).

- *The spatiotemporal criterion*

    ∗ (1-A) Impure sets are abstract, as $\{Earth\}$ is a set and all sets (whether pure or impure) exist outside spacetime. (1-C) Impure sets are concrete: e.g. $\{Earth\}$ exists only in space where Earth exists, at any time at which $\{Earth\}$ exists (whether eternally or when Earth exists).

    ∗ (2-A) Abstract artifacts are abstract: e.g. the novel *Robinson Crusoe* can be considered as a possible pattern of words and it exists outside spacetime, just as pure sets (e.g. the empty set $\emptyset$) do. (2-C) Abstract artifacts are concrete: e.g. the novel *Robinson Crusoe* comes into being when it is created by Daniel Defoe (or it exists eternally) and, at any time at which it exists, the novel exists only in space where any of its "tokens" — such as a copy of the novel — exists.

- *The causal criterion*

    ∗ (1-A) Impure sets are abstract, as $\{Earth\}$ is causally inefficacious, as distinct from Earth (which is causally efficacious). (1-C) Impure sets are concrete, as $\{Earth\}$ comes into being as a result of the birth of Earth.

    ∗ (2-A) Abstract artifacts are abstract, as the novel *Robinson Crusoe* is causally inefficacious, as distinct from its tokens (e.g. a copy of the novel) which are causally efficacious. (2-C) Abstract artifacts are concrete, as the novel *Robinson Crusoe* comes into being as a result of Daniel Defoe's act of creation.

Arguably, impure sets and abstract artifacts can be equally plausibly considered as abstract or concrete by any of the three criteria.

There are some possible interpretations of this argument. One interpretation is that all the three criteria must be further developed to classify clearly such borderline cases. Each criterion may nevertheless have its own difficulty. Consider for instance the spatiotemporal criterion, as it is accepted by BFO, DOLCE and GFO. Falguera et al. [2] argue that, in order to explain the difference between pure and impure sets while thinking that all sets are abstract by the spatiotemporal criterion, one will need to elaborate the notion of some distinctive relation of impure sets towards spacetime. As Falguera et al. also say, the ontological status of spacetime, especially whether it is abstract or concrete, can also be problematic for the spatiotemporal criterion. As is indicated by our survey, there are indeed divergent views on space and time among BFO, DOLCE and GFO.

Another interpretation — which we will explore below — is that it may be reasonable to question the shared assumption by the three criteria that any entity is either abstract or concrete but not both. All these three criteria are committed to the following two views of the abstract/concrete distinction:

The exhaustive view of abstracta and concreta
Any entity is either abstract or concrete.

The exclusive view of abstracta and concreta
No entity is both abstract or concrete.

These two views together would conflict with the previous analysis that impure sets and abstract artifacts can be equally plausibly construed as abstract or concrete according to any of the three criteria. There are at least two possible options to deal with such problematic entities. One option is to think that they are *neither* abstract *nor* concrete because they do not clearly belong to either category.[6] The other option is to think that they are *both* abstract *and* concrete because they can be understood in either way. The first and second options will lead, respectively, to the following views of the abstract/concrete distinction that are opposed to the exhaustive and exclusive ones:

The non-exhaustive view of abstracta and concreta
There is some entity that is neither abstract nor concrete.

The non-exclusive view of abstracta and concreta
There is some entity that is both abstract and concrete.

In what follows we will develop a formal framework for representing abstract and concrete entities which can make room for the non-exhaustive and non-exclusive views and which can be therefore more general than these three existing criteria.

3.2. Basic Idea: Concretization, (Purely) Abstract and (Purely) Concrete

We will elaborate a *relational* account of abstract and concrete entities to develop a general framework for representing them formally. Recall that a generically dependent continuant in BFO — which is arguably causally abstract — exists only if it is "concretized" in some other kind of entity and, in Sowa's [23] ontology (discussed by GFO), a physical entity "represents" or "characterizes" an abstract entity. To put it generally, a concrete entity can bear some special relation towards an abstract entity.

We propound the view that the abstract/concrete distinction can be defined in terms of the relation of *concretization* (which should not be conflated with concretization in BFO, though). The core idea is that being abstract (respectively: concrete) amounts to being concretized by (respectively: concretizing) some other entity. To articulate this idea, we introduce the four notions: abstract, purely abstract, concrete and purely concrete. We define them by means of the concretization relation as follows:

x is purely abstract if and only if there is no y such that x concretizes y.
x is concrete if and only if there is some y such that x concretizes y.
x is abstract if and only if there is some y such that y concretizes x.
x is purely concrete if and only if there is no y such that y concretizes x.

Informally, an abstract entity is something that is "concretizable" ("can be made concretized") and a purely abstract entity is something that that cannot concretize any other entities; whereas, a concrete entity is something that can concretize some other entity and a purely concrete entity is something that is not further concretizable.

To illustrate our relational account of abstracta and concreta, we will assume that there are at least two subtype relations of concretization. One is the so-called "token-

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[6]This line of approach is suggested by Falguera et al. [2]: "we should be open to the possibility that the best sharpening of [the abstract/concrete distinction] will entail that some objects are neither abstract nor concrete. [...] The main constraint on an account of the distinction is that it draws a philosophically significant line that classifies at least many of the standard examples in the standard ways. It is not a constraint that everything be shoehorned into one category or the other."

type" relation. For example, the novel *Robinson Crusoe* is a type-level entity and one of its tokens is my copy of the novel (where, to wit, the term "copy" here would refer to "physical encoding" and "ink pattern" in Sowa's and BFO's parlance, respectively). In effect, the token-type relation would correspond to Sowa's characterization/representation relation and BFO's concretization relation. Relatedly, the other is the instantiation relation between a particular and a universal. In GFO, an individual can be said to characterize or represent (in Sowa's sense) a category. We can generalize this statement in such a way that a particular instantiates — and thus concretizes — a universal.[7]

Given this assumption about concretization, we will use the illustrative example in which the novel *Robinson Crusoe* concretizes the universal *Literary Work* and and it is concretized by my copy of the novel. We can say that the novel *Robinson Crusoe* is both abstract and concrete, the universal *Literary Work* is abstract and my copy of the novel is concrete. Assuming that there is no other concretizaton relation, the universal *Literary Work* is purely abstract and my copy of the novel is purely concrete. We will formalize below this relational account of the abstract/concrete distinction.

3.3. Formal Framework

In order to provide a formal framework for concretization, we must first specify our language. Let $x_1, x_2, \ldots$ be a countably infinite supply of *variables*, and *concretizes* be a binary predicate. Our first-order language is then specified by the next clauses.[8]

$$\varphi ::= concretizes(x, y) \mid x = y \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi) \mid \forall x \varphi \mid \exists x \varphi \mid \bot.$$

$concretizes(x, y)$ is read as "x concretizes (makes concrete) y". Parentheses will be abbreviated when there is no fear of confusion. $\varphi \leftrightarrow \psi$ and $\neg \varphi$ will abbreviate $(\varphi \rightarrow \psi) \wedge (\psi \rightarrow \varphi)$ and $\varphi \rightarrow \bot$, respectively. We shall base our formalization on classical first-order logic (with equality) [24] to make it readily available to existing ontologies.

We shall axiomatize the relation of concretization by the following schemata.

$$\forall x \neg concretizes(x, x) \tag{Ax1}$$

$$\forall x, y, z(concretizes(x, y) \wedge concretizes(y, z) \rightarrow concretizes(x, z)) \tag{Ax2}$$
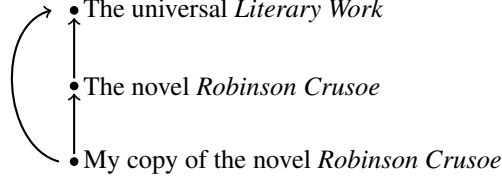
$$\forall x \exists y(concretizes(x, y) \vee concretizes(y, x)) \tag{Ax3}$$

$$\forall x \exists y \forall z, u(concretizes(z, x) \rightarrow (concretizes(y, x) \wedge \neg concretizes(u, y))) \tag{Ax4}$$

An intuitive explanation of the axioms is as follows. (Ax1) says that the concretization relation is *irreflexive*: i.e. nothing concretizes itself. (Ax2) says that the concretization relation is *transitive*: e.g. my copy of the novel *Robinson Crusoe* concretizes the novel, and the novel in turn concretizes the universal *Literary Work*; and this necessitates that my copy of the novel *Robinson Crusoe* concretizes the universal *Literary Work*. (Ax3) is meant to ensure that there is no "isolated entity": anything must either concretize something or be concretized. Finally, (Ax4) states that anything may be concretized, to such a degree that it cannot be further concretized (see Section 4.1 for discussion).

In relation to the last point, we define a few more unary predicates.

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[7]Note that *universals* are a subtype of *categories* in GFO [22], though.

[8]Here we keep ourselves to a simple language, but we may also include e.g. constants in the usual way.

Figure 1. Example of concretization (represented by an arrow)

- *purely_abstract*(x) ≜ ∀y¬*concretizes*(x,y).
- *abstract*(x) ≜ ∃y(*concretizes*(y,x)).
- *concrete*(x) ≜ ∃y(*concretizes*(x,y)).
- *purely_concrete*(x) ≜ ∀y¬*concretize*(y,x).

It is not difficult to observe that these predicates correspond to the notions of being abstract and purely abstract (as well as concrete and purely concrete) in Section 3.2.

It is straightforward to see that a structure represented by the diagram (see Figure 1) satisfies (Ax1)–(Ax4). As far as this diagram is concerned, the universal *Literary Work* is purely abstract, my copy of the novel *Robinson Crusoe* is purely concrete and the novel *Robinson Crusoe* is both abstract and concrete.

As some consequences of the above definitions and axioms, we have:

$$\forall x,y(concretizes(x,y) \rightarrow \neg concretizes(y,x)) \tag{Cn1}$$

$$\forall x(abstract(x) \vee concrete(x)) \tag{Cn2}$$

$$\forall x \neg (purely\_abstract(x) \wedge purely\_concrete(x)) \tag{Cn3}$$

$$\forall x \exists y(abstract(x) \rightarrow (concretizes(y,x) \wedge purely\_concrete(y))) \tag{Cn4}$$

We remark that the concretization relation is *asymmetric* [(Cn1); by (Ax1), (Ax2)]. We also note the contrast between the notions of being abstract and concrete, on the one hand, and those of being purely abstract and purely concrete, on the other hand. The former pair of notions allow a *glut* but not a *gap*: something may be both abstract and concrete, but nothing is neither abstract nor concrete [(Cn2); by (Ax3)]. The latter pair allows a gap but not a glut: something may be neither purely abstract nor purely concrete, but nothing is both purely abstract and purely concrete [(Cn3); by (Ax3)].

4. Discussion

4.1. Evaluation

We will elucidate and evaluate our relational account of abstract and concrete entities by discussing the axioms and consequences in our formal framework.

As for (Ax1) and (Ax2): The irreflexivity and transitivity of concretization may be motivated by analogy with those of proper parthood in mereology [25] — that is, nothing is proper part of itself (irreflexive); and if x is proper part of y and y is proper part of z, then x is proper part of z (transitive). The idea is that, when an entity x concretizes another entity y, x could be seen as having a "(proper) part", in some sense, of y. To illustrate

this point with the example in Figure 1, my copy of the novel *Robinson Crusoe* has the "Crusonian part" of the novel and the novel *Robinson Crusoe* has the "literary work-ness part" of the universal *Literary Work*. And indeed, my copy of the novel *Robinson Crusoe* has the "literary work-ness part" of the universal *Literary Work*. We also remark that an implausible possibility of loops — i.e. $x_1, \ldots, x_n$ such that *concretizes*(x_2, x_1), *concretizes*(x_3, x_2), ..., *concretizes*(x_1, x_n) — is avoided thanks to (Ax2) (and (Ax1)).

As for (Cn2) and (Cn3) [by (Ax3)]: As said in Section 3.1, the three existing criteria (epistemic, spatiotemporal and causal) for the abstract/concrete distinction presuppose the exhaustive and exclusive views; by contrast, our relational account of abstract and concrete entities intends to accommodate the non-exhaustive and the non-exclusive views. In our framework, the notions of being abstract and concrete accommodate the exhaustive and non-exclusive views, because any entity is either abstract or concrete and there are "glutty entities" — which are both abstract and concrete. And the notions of being purely abstract and purely concrete accommodate the exclusive and non-exhaustive views, because no entity is both purely abstract and purely concrete and there are "gappy entities" — which are neither purely abstract nor purely concrete.

As for (Ax4): Quite importantly, this axiom does not *ipso facto* exclude the possibility that a process of concretization never reaches to an entity which cannot concretize any other entities. To illustrate this possibility, consider atomism in mereology [25]. Atomism is the thesis that everything is composed of *atoms*: entities with no proper parts. The "atomist axiom" that simply says that everything *has* atomic parts — see e.g. the axiom (A.6) [25, p. 42] — does not imply that everything *decomposes into* atoms; to wit, it is *composed of* atoms. In the atomist system: "The decomposition process goes on forever" [25, p. 147]. (Ax4) can be understood by analogy with the atomist axiom.

As for (Cn4) [by (Ax4)]: It states that anything abstract is concretized by some purely concrete entity. But we do not have the following counterpart statement:

$$\forall x \exists y (concrete(x) \rightarrow (concretizes(x, y) \wedge purely\_abstract(y)))$$

That is to say, anything concrete concretizes some purely abstract entity. The issue here is the difference between a process of being concretized and a process of concretizing. On the one hand, the former process seems to have some "end point". For instance, it may not be clear whether and how the planet Earth can be further concretized (hence (Ax4) and (Cn4)). On the other hand, the latter process might not seem to have such an end point, as a mental process of abstraction could endlessly continue in theory.

4.2. A Concretization-Based Reinterpretation of the Spatiotemporal Criterion

It will be desirable to consider how existing criteria for distinguishing between abstract and concrete entities can be reconstrued within our formal framework for representing them. For space reasons, we will briefly illustrate this point with the spatiotemporal criterion (as it is widely adopted by BFO, DOLCE and GFO) and the impure set $\{Earth\}$, as distinct from pure sets (e.g. the empty set $\emptyset$) and Earth — which are standardly taken, respectively, as spatiotemporally abstract and concrete (see Section 2.1). For the sake of simplicity, we will assume the notion of spacetime, rather than space and time.

One simple, concretization-based way of reinterpreting the spatiotemporal criterion is to think that being spatiotemporally abstract (respectively: spatiotemporally concrete) amounts to being concretized by (respectively: concretizing) spacetime:

> *x* is spatiotemporally abstract if and only if spacetime concretizes *x*.
>
> *x* is spatiotemporally concrete if and only if *x* concretizes spacetime.

Note that there are some entities that are neither spatiotemporally abstract nor spatiotemporally concrete (non-exhaustive) — e.g. spacetime, in harmony with one construal of GFO's view of space and time (see Section 2.2.3); but no entity is both spatiotemporally abstract and spatiotemporally concrete (exclusive).

One possible way of spelling out the notions of "spacetime concretizing something" and "being concretized by spacetime" would be to interpret concretization as some kind of "manifestation" relation (which may be compared to Sowa's [23] "representation" relation). For instance, material bodies may be said to "concretely manifest" spacetime in the sense that, given modern physics, the motion of a material body (which we can perceive) can be analyzed as its trajectory through spacetime. We can think that Earth is spatiotemporally concrete because it concretizes, in this sense, spacetime. In addition, spacetime may be said to "concretely manifest" the empty set $\emptyset$, as it could be considered as an "image" of such "eternal" entities, as in mathematics.[9] Thus the empty set $\emptyset$ is spatiotemporally abstract because it is concretized, in this sense, by spacetime.

By contrast, whether the impure set $\{Earth\}$ is either spatiotemporally abstract or spatiotemporally concrete will depend on more specific manifestation-based characterizations of concretization, e.g. *vis-à-vis* set membership. Moreover, the difficulty of providing such characterizations may motivate us to think that $\{Earth\}$ is neither spatiotemporally abstract nor spatiotemporally concrete (see Section 3.1) — which constitutes a third construal of impure sets in virtue of the non-exhaustive view in our framework.

5. Conclusion

This paper aimed to explore a foundational basis for an ontology of abstract and concrete entities. For this purpose, we presented three existing criteria (epistemic, spatiotemporal and causal) for their distinction and illustrated them with three major upper ontologies: BFO, DOLCE and GFO. After examining the shared assumption by the three criteria that any entity is either abstract or concrete but not both, we developed an alternative and more general framework for formally representing abstracta and concreta — especially a relational account of them, whereby the four key notions (viz. abstract, purely abstract, concrete and purely concrete) are defined in terms of the concretization relation.

In the future we will further elaborate our formal framework for characterizing abstract and concrete entities. We plan to reformulate and compare these three (and other) existing criteria within this framework and to develop a way of integrating our concretization-based approach into various upper ontologies. It will also be valuable to investigate the relationship between our proposal and other formal theories of the abstract/concrete distinction, such as Zalta's modal logical approach [27] and type-theoretical approach [28] to abstract entities.

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[9]To elucidate this statement, consider Plato's teleological explanation of the existence of the universe and of its general features in his *Timaeus*. For Plato: "Time itself came into being together with [...] celestial movements as an 'image of eternity' " [26]. This platonic view of time as an "image of eternity" might be generalized into the notion of spacetime.

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